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TORSIONAL STRENGTH OF ALUMINUM-ALLOY ROUND TUBING

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SUMMARY

An analysis has been made of existing data on aluminum-alloy tubing with a wide range of plastic properties in order to establish a useful empirical relationship between tensile yield and ultimate strengths, diameter-thickness ratios, and torsional strengths within the range of plastic buckling. The results indicate that the upper limit of torsional strength for a round tube is determined by the shear strength of the material. The shear strength of the heat-treated wrought aluminum alloys in torsion may be taken conservatively at about 65 percent of the tensile strength. For round tubes with diameter-thickness ratios ranging from about 10 to 50 or 60 and with lengths greater than about 3 diameters, failures in torsion may be expected by plastic buckling at stresses below the shear strength of the material.

INTRODUCTION

Although a number of investigations of the torsional strength of aluminum-alloy round tubing have been made, apparently no general method for predicting such strengths within the range of plastic buckling failures has ever been proposed. The importance of this type of behavior from the standpoint of aircraft design is indicated by the fact that the torsional moduli of rupture given for aluminum-alloy tubing in figure 5-7 of reference 1 are limited entirely to cases involving plastic action.

It is generally recognized from the results of previous work (references 2 and 3) that torsional strengths within the range of plastic buckling are dependent mainly upon the yield and ultimate strengths of the material in shear, which give a measure of the extent of the plastic range, and upon the diameter-thickness ratios of the tubing. In view of the approximate affinity that exists between the shear and tensile properties of certain aluminum alloys, attempts have been made to correlate torsional strengths with yield and ultimate strengths in tension rather than with the corresponding properties in shear. Within the range of tests of any one aluminum alloy and temper, however, there has not been sufficient variation in ratios of tensile yield to ultimate strength to show clearly the significance of this factor. As a result, tensile strength has been the only property included thus far in empirical formulas for torsional strength.

The purpose of this report is to show from an analysis of existing data on aluminum-alloy tubing with a wide range of plastic properties that a useful empirical relationship between tensile yield and ultimate strengths, diameter-thickness ratios, and torsional strengths within the range of plastic buckling can be established,

TEST RESULTS

The experimental data used in this analysis were obtained from tests made at the National Bureau of Standards (reference 2), at Wright Field (reference 3), and at the Aluminum Research Laboratories (references 4, 5, and 6, and unpublished data).

ANALYSIS OF RESULTS

Although this analysis is concerned primarily with the torsional strength of tubing within the range of plastic buckling, the extent of this range necessarily involves some consideration of the boundary cases of plastic shear and elastic buckling. Three basic types of action have been considered: (1) tubes that fail by plastic yielding without buckling, for which torsional strengths are limited by the shear strength of the material; (2) tubes that fail by plastic buckling at stresses between the limits of plastic shear and elastic buckling, which probably include most of the sizes common in aircraft construction; and (3) tubes that fail by elastic buckling, for which torsional strengths may be estimated by any one of several existing theories. (See references 6 and 7.)

Plastic Shear

Ultimate shear strengths in torsion may be determined from tests of specimens with a variety of proportions, provided that failure occurs by plastic yielding and fracture in shear, rather than by buckling. (See reference 4.) Figure 1 shows the results of a number of tests of this type in which diameter-thickness ratios (D/t) were varied from 2 to 18. Each specimen had a reduced section of length equal to about 1/2 diameter (L/D = 0.5) that minimized the possibility of any type of failure other than shear fracture on a section normal to the axis of twist.

The computation of shear strengths from such tests as those shown in figure 1 involves some assumption regarding the type of stress distribution obtained at failure. Two sets of shear-strength values have therefore been included: extreme-fiber stresses computed on the assumption of elastic action up to the point of failure, often called moduli of failure or rupture; and stresses computed on the assumption of a uniform stress distribution. The differences between the two sets of values decrease with increasing D/t ratios, ranging from a maximum of $33\frac{1}{3}$ percent for a solid round bar having a D/t of 2 to approximately 10 percent for a D/t of 10, or only 2 percent for a D/t of 50. Although moduli of failure have been used in many cases as a measure of shear strengths in torsion and might also have been used here, the assumption of a uniform distribution of shear stress is believed to be somewhat more consistent with the actual behavior obtained at failure in a ductile material and has been used throughout this analysis. The uniformity in shear strengths shown in figure 1 on the basis of this assumption for specimens having a wide range of D/t ratios would appear to validate this procedure.

The computations for moduli of failure referred to were made by means of the familiar torsion formula

$$S_1 = \frac{T}{I_p} r \qquad (1)$$

where

S_1 modulus of failure, pounds per square inch

T torque, inch-pounds

r radius to extreme fiber, inches

I_p polar moment of inertia, inches^4

Shear stresses corresponding to a uniform distribution were computed by means of the relationship

$$S_2 = \frac{3T}{2\pi\,(r_1{}^3 - r_2{}^3)} \qquad (2)$$

where

S_2 uniform shear stress, pounds per square inch

r_1 outside radius, inches

r_2 inside radius, inches

It should be pointed out that for specimens having a D/t ratio greater than about 10, the shear stresses computed by equation (2) are for all practical purposes the same as the mean-fiber stresses that may be obtained by equation (1), where r equals the radius to the mean fiber rather than to the extreme fiber, or the average stresses that may be obtained by the approximate formula

$$S_3 = \frac{T}{2\pi r^2 t} \qquad (3)$$

where

S_3 average shear stress, pounds per square inch

r radius to median fiber, inches

t wall thickness, inches

Although the ultimate shear strength of a material represents the upper limit of stress that may be developed in torsion, the determination of this property requires a special test. It is convenient from the standpoint of a general analysis to express shear strengths in terms of tensile strengths. From the data already considered in figure 1 and from the ratios of shear to tensile strength shown in figure 2, it is believed that the shear strengths for the aluminum alloys in all but the annealed or-O temper may be taken conservatively at about 65 percent of the tensile strengths. Only 2 of the 11 materials considered in this range indicated ratios below this value, and the differences in these cases (61S-T and 51S-T) were only 2 to 3 percent. No attempt has been made to include the annealed tempers in this generalization as torsion tests other than those to determine shear strengths have not been made of these materials.

Although figures 1 and 2 indicate the maximum shear strengths to be expected in specimens designed for plastic shear failures, the limits for this type of action involve a consideration of both diameter-thickness ratios (D/t) and length-diameter ratios (L/D). As is shown in figure 3, 17S-T specimens with a D/t of 10 were the only ones for which the shear strength of the material could be developed in tubes with a variety of lengths, ranging from 0.2 to 6 times the diameter. For a D/t of 14 the strength for an L/D of 3 was about 8 percent less than for an L/D of 0.5. For a D/t of 18 the corresponding decrease was about 20 percent.

The results shown for the 61S-T tubing in figure 3 indicate that variations in length ranging from about 5 to 28 diameters had no effect upon torsional strengths for D/t ratios up to 40 and only a slight effect for ratios as high as about 60. In no instance, however, were the strengths obtained equal to the shear strength of the material, or the value determined using a D/t of 10. (See fig. 2.) Increases in length beyond a few diameters, or that length necessary to permit plastic buckling failures, apparently have no significant effect upon torsional strengths within this range. If the curves shown for the 61S-T tubing had included L/D ratios as low as 0.5, it is believed that length effects similar to those found for the short specimens of 17S-T would also have been observed.

It is concluded from these observations that a D/t of 10 is about the highest that can be used to determine shear strengths in torsion unless restrictions are placed upon the length of specimen used. This limiting ratio of D/t has been selected therefore to mark the beginning of the range of plastic buckling failures.

Plastic Buckling

It was demonstrated in tests made at the National Bureau of Standards (reference 2) on 17S-T tubing that an approximately linear relationship exists between ratios of torsional strength to tensile strength and the reciprocal of D/t within the plastic buckling range. Similar observations have since been made for 24S-RT tubing at Wright

Field (reference 3) and for 61S-T tubing at this laboratory (reference 5). The significant point noted from a comparison of these straight-line relations for the different alloys was that the slopes varied roughly with the extent of the plastic range of the material, or the spread between tensile yield and ultimate strengths. For 61S-T tubing with a ratio of tensile yield to ultimate strength of 0.89, the slope was considerably flatter than for 17S-T tubing, with a ratio of tensile yield to ultimate strength of 0.72. This observation suggested the possibility of establishing a relationship between slopes and ratios of tensile yield to ultimate strength that could be used as a basis for predicting torsional strengths.

Figures 4 and 5 show to what extent linear relationships may be obtained in the range of plastic buckling by plotting ratios of torsional strength to tensile strength against the reciprocal of D/t. Data from eight different series of tests of aluminum-alloy tubing have been used. The average tensile yield and ultimate strengths indicated are typical for commercial tubing. Ratios of tensile yield to ultimate strength ranged from 0.61 to 0.89, values of D/t ranged from about 10 to 50, and values of L/D ranged from about 4 to 56. Differences in slope, which are the results of principal interest, are indicated by the values of K shown. No attempt has been made to include detailed information on L/D ratios as it was quite evident that this factor had little bearing upon torsional strengths within the range of plastic buckling.

Figure 6 shows the relationship between slopes (values of K from figs. 4 and 5) and ratios of tensile yield to ultimate strength. Two fairly definite ranges of action seem to be indicated: one for tensile yield-ultimate strength ratios from about 0.6 to 0.75, for which K remains fairly constant; the other for tensile yield-ultimate strength ratios greater than 0.8 for which there is a marked decrease in K for increasing strength ratios. The curve for ratios greater than 0.8 has been extrapolated to $K = 0$ for a tensile yield-ultimate strength ratio of unity, which would seem to be an approximate limit for a material having no plastic range.

If the curve in figure 6 is accepted as indicating a satisfactory relationship between K and ratios of tensile yield to ultimate strength, a general expression may be written for torsional strength that recognizes, to some degree at least, the significance of the plastic range of a material. Such an expression has been written as

$$\tau = \sigma\left[0.65 - K\left(0.10 - \frac{t}{D}\right)\right] \tag{4}$$

where

τ torsional strength based on a uniform distribution of shear stress (not to exceed 0.65σ), pounds per square inch

σ tensile strength, pounds per square inch

K factor from figure 6, depending upon ratio of tensile yield to ultimate strength

D outside diameter, inches

This formula was derived simply as the equation for a series of straight lines of varying slope K that have a common origin at the point $t/D = 0.1$ and $\tau/\sigma = 0.65$, the assumed junction of the ranges of plastic buckling and plastic shear.

Figures 7 and 8 show the same torsional strengths as those used in figures 4 and 5, plotted in a somewhat more conventional manner against ratios of D/t. Computed curves for plastic buckling obtained by means of equation (4) and by the curve in figure 6 have also been included for comparison. The close agreement in most cases is believed to demonstrate the general applicability of the method of predicting torsional strengths. Although the computed curves shown for several of the alloys are substantially the same as have been derived separately before, by use of the method of plotting shown in figures 4 and 5, equation (4) differs from other empirical expressions that have been proposed in that it takes into account differences in the plastic properties of materials, which clearly have a significant bearing upon torsional strengths. In the case of the 61S-T tubing with a ratio of tensile yield to ultimate strength of 0.89, the computed torsional strength for a D/t of 50 is only about 22 percent less than for a D/t of 10. In the case of the 51S-W tubing, however, with a ratio of tensile yield to ultimate strength of 0.61, the corresponding decrease in torsional strengths is about 50 percent.

Figure 9 gives a comparison between torsional strengths computed by the method proposed here for 24S-RT, 24S-T, and 17S-T tubing with specified minimum properties, and torsional moduli of rupture for the same material as given in figure 5-7 of reference 1. The agreement between the torsional strengths indicated by the two sets of curves for values of D/t from about 10 to 30 is actually closer than shown, as the values given in reference 1 are expressed in terms of extreme-fiber stresses; whereas those computed by the proposed method are based upon the assumption of a uniform distribution of shear stress at failure. Figure 1, previously considered, indicates the influence of method of stress computation upon the interpretation of tests of this type. For D/t ratios greater than 30, on the other hand, the differences between torsional strengths are actually greater than those indicated in figure 9, although the amount is not significant, ranging from about 3 percent for a D/t of 30 to 2 percent for a D/t of 50. It will be noted that for D/t ratios less than 11, covering the range of plastic shear failures, the curves shown in reference 1 indicate uniform torsional strengths. Such a result is obviously not consistent with the computation of torsional strengths as extreme-fiber stresses as implied and suggests that a uniform distribution of stress must have been assumed.

The curves for torsional strength (reference 1) shown in figure 9 were apparently obtained by substituting specified minimum values of tensile strength in experimentally derived formulas without regard to tensile yield-ultimate strength ratios. Although the curves computed by the method proposed here for plastic buckling admittedly do not indicate appreciably different values of torsional strength for the alloys considered, the need for recognizing ratios of tensile yield to ultimate strengths is believed to be none the less important.

Elastic Buckling

The range of elastic buckling is of interest in this investigation only as it limits the applicability of the proposed formula for plastic buckling. The transition between the ranges of plastic and elastic buckling is less easily defined than that between the ranges of plastic buckling and plastic shear, since L/D ratios as well as D/t ratios become involved. As has been previously shown, the upper limit for plastic buckling is the shear strength of the material, which apparently may be developed in tubes having D/t ratios of about 10 regardless of length. It has also been demonstrated that except in the case of very short tubes, with L/D ratios less than about 3, length does not have a significant bearing upon torsional strengths for D/t ratios as high as 50 or 60. For larger D/t ratios, however, buckling may occur at stresses within the elastic range and the length factor becomes involved. A family of curves, one for each value of L/D, is then necessary to show the variation in torsional strength with D/t; whereas, in the plastic range, single curves such as those shown in figures 7 and 8 are sufficient.

The computed curves for plastic buckling in figures 7 and 8 were stopped at a D/t of 50, as this limit covered the range of tests considered. It was not intended, however, that this ratio should mark the end of the plastic buckling range, which is presumably reached for a particular case when the torsional strength computed by the proposed method for plastic buckling exceeds that indicated by one of the existing theories for elastic buckling. (See references 6 and 7.) The reasonableness of this procedure as a practical expedient for covering the entire range of torsion failures, both elastic and plastic, is supported by the experimental data and computed curves shown in figure 10. Although the agreement between the test and theoretical values (based on analysis by R. G. Sturm, summarized in reference 6) of torsional strength in the elastic range is not particularly good in some cases, the influence of L/D and the need for a family of curves to describe variations in torsional strength with D/t is clearly indicated.

CONCLUSIONS

The foregoing analysis of test results from various sources on material representative of commercial production is believed to justify the following conclusions regarding the torsional strength of wrought aluminum-alloy round tubing:

1. The upper limit of torsional strength for a round tube is determined by the shear strength of the material. Specimens for the determination of this property may have a variety of proportions, provided that failure occurs by plastic yielding or fracture in shear without buckling. Unless restrictions are placed upon the length of the specimen used for this purpose, however, diameter-thickness ratios should not exceed about 10.

2. The shear strength of the heat-treated wrought aluminum alloys in torsion may be taken conservatively at

about 65 percent of the tensile strength. This value is based upon the assumption of a uniform distribution of shear stress at failure, which seems to be a reasonable procedure in the case of ductile materials of the kind considered.

3. For round tubes having diameter-thickness ratios ranging from about 10 to 50 or 60 and lengths greater than about 3 diameters, failures in torsion may be expected by plastic buckling at stresses below the shear strength of the material. Torsional strengths within this range may be predicted by means of the empirical formula:

$$\tau = \sigma\left[0.65 - K\left(0.10 - \frac{t}{D}\right)\right] \tag{4}$$

where

τ torsional strength based on a uniform distribution of shear stress (not to exceed 0.65σ), pounds per square inch

σ tensile strength, pounds per square inch

K factor based upon ratio of tensile yield to ultimate strength, given in figure 6

t wall thickness, inches

D outside diameter, inches

4. The agreement shown in figures 7 and 8 between torsional strengths obtained in tests of aluminum-alloy round tubing at several different laboratories and the corresponding strengths computed by means of equation (4) is believed to establish this equation as a reasonably satisfactory method of predicting torsional strengths. The inclusion of the ratio of tensile yield to ultimate strength as a significant factor makes the method applicable to alloys having a considerable range of plastic properties.

5. Although plastic buckling failures may be expected in aluminum-alloy round tubes with diameter-thickness ratios as high as 50 or 60 without regard to length effects, the limit of applicability of equation (4) for plastic buckling is presumably reached when torsional strengths so computed exceed theoretical values for elastic buckling.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Pa., September 15, 1942.

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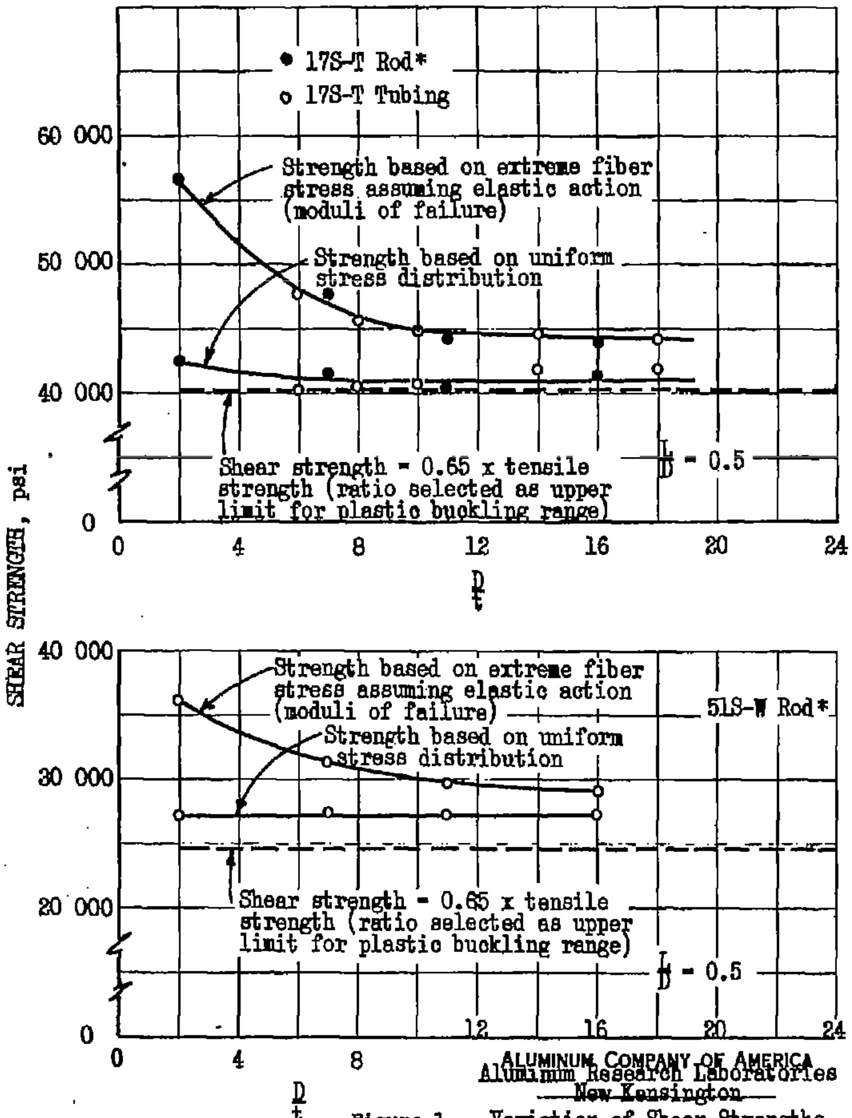

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Figure 1.- Variation of Shear Strengths in Torsion with $\frac{D}{t}$

PHYSICAL TEST No. 101829-GG
KIND OF METAL As Noted

* $\frac{D}{t}$ ratios greater than 2 obtained by boring out center of rod.

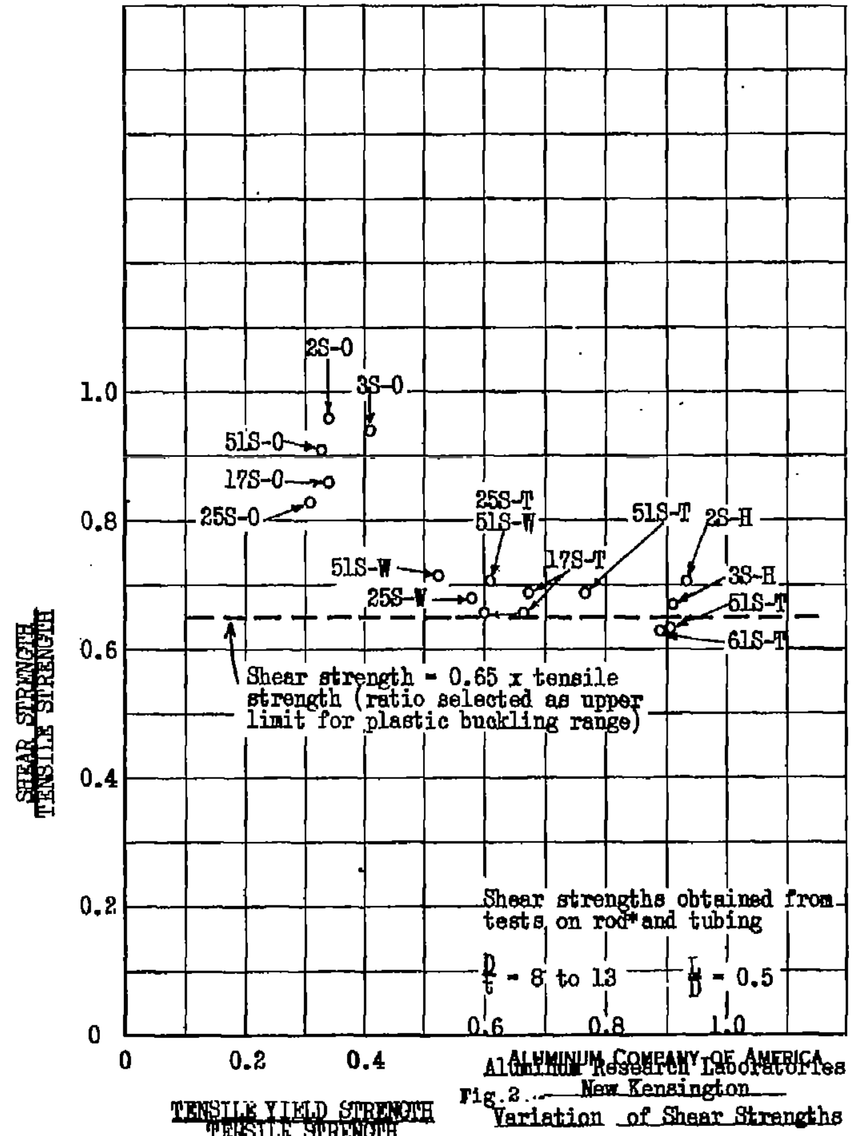

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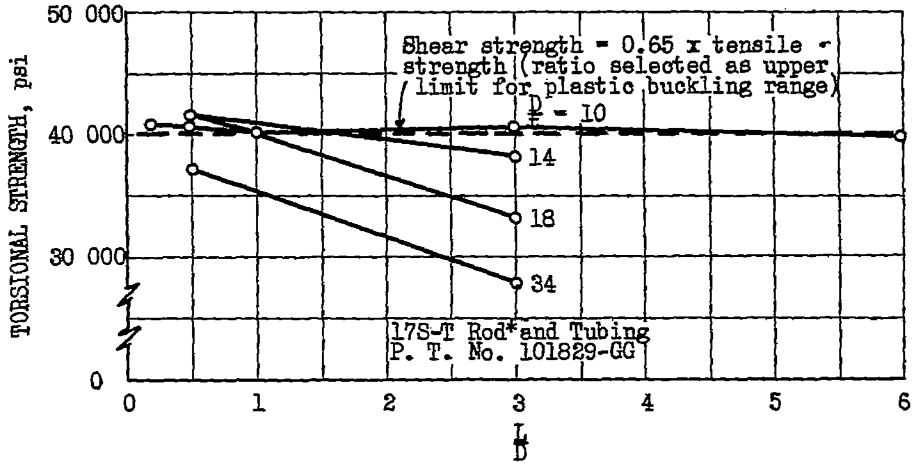
Fig. 2.- Variation of Shear Strengths in Torsion with Tensile Properties

PHYSICAL TEST No. 101829-GG, 051730-A, 102741-A
KIND OF METAL As Noted

* $\frac{D}{t}$ ratios greater than 2 obtained by boring out center of rod.



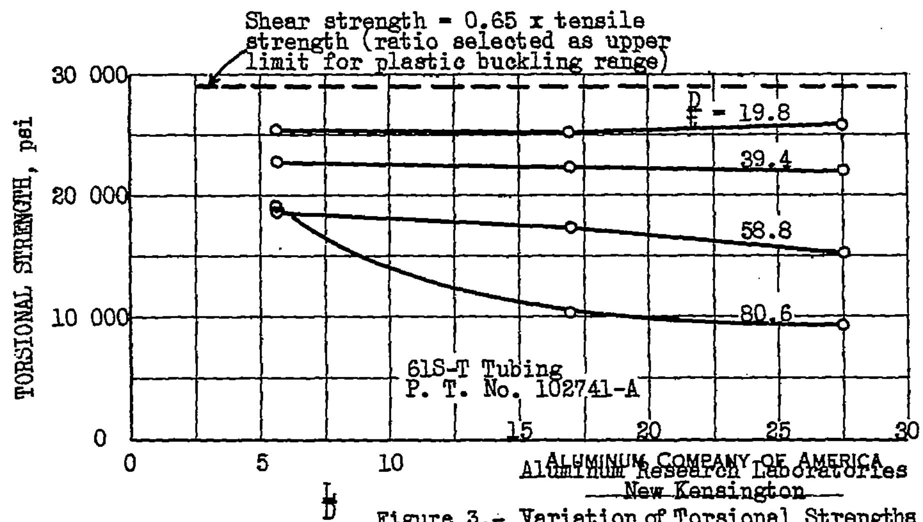


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Figure 3.- Variation of Torsional Strengths with $\frac{L}{D}$

* $\frac{D}{t}$ ratios greater than 2 obtained by boring out center of rod.

PHYSICAL TEST No. As Noted
KIND OF METAL As Noted
PLOTTED DATE APPROVED DATE

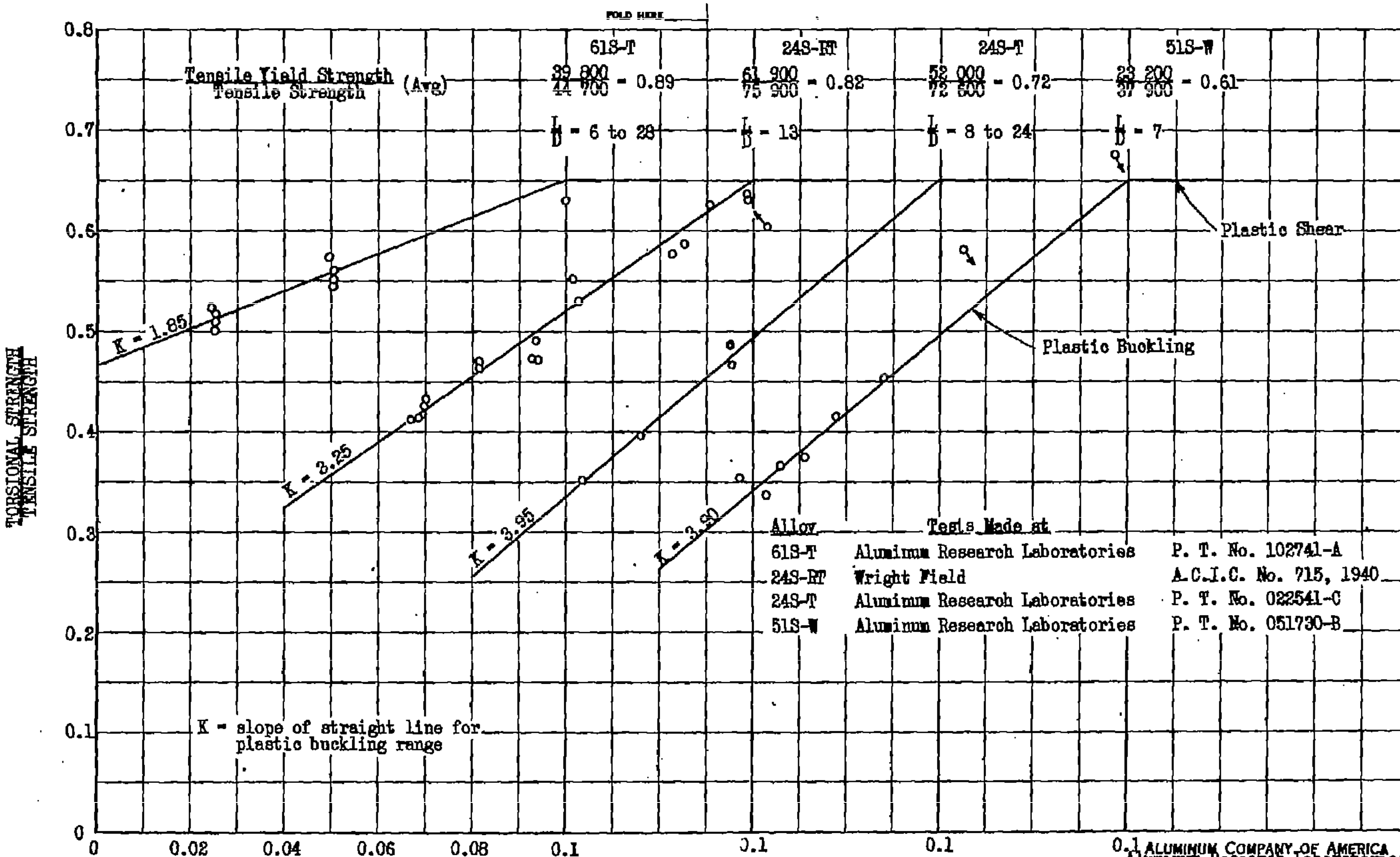

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Figure 4.- Variations of Ratio of Torsional Strength to Tensile Strength with $\frac{t}{D}$

PHYSICAL TEST NO. As Noted
KIND OF METAL As Noted

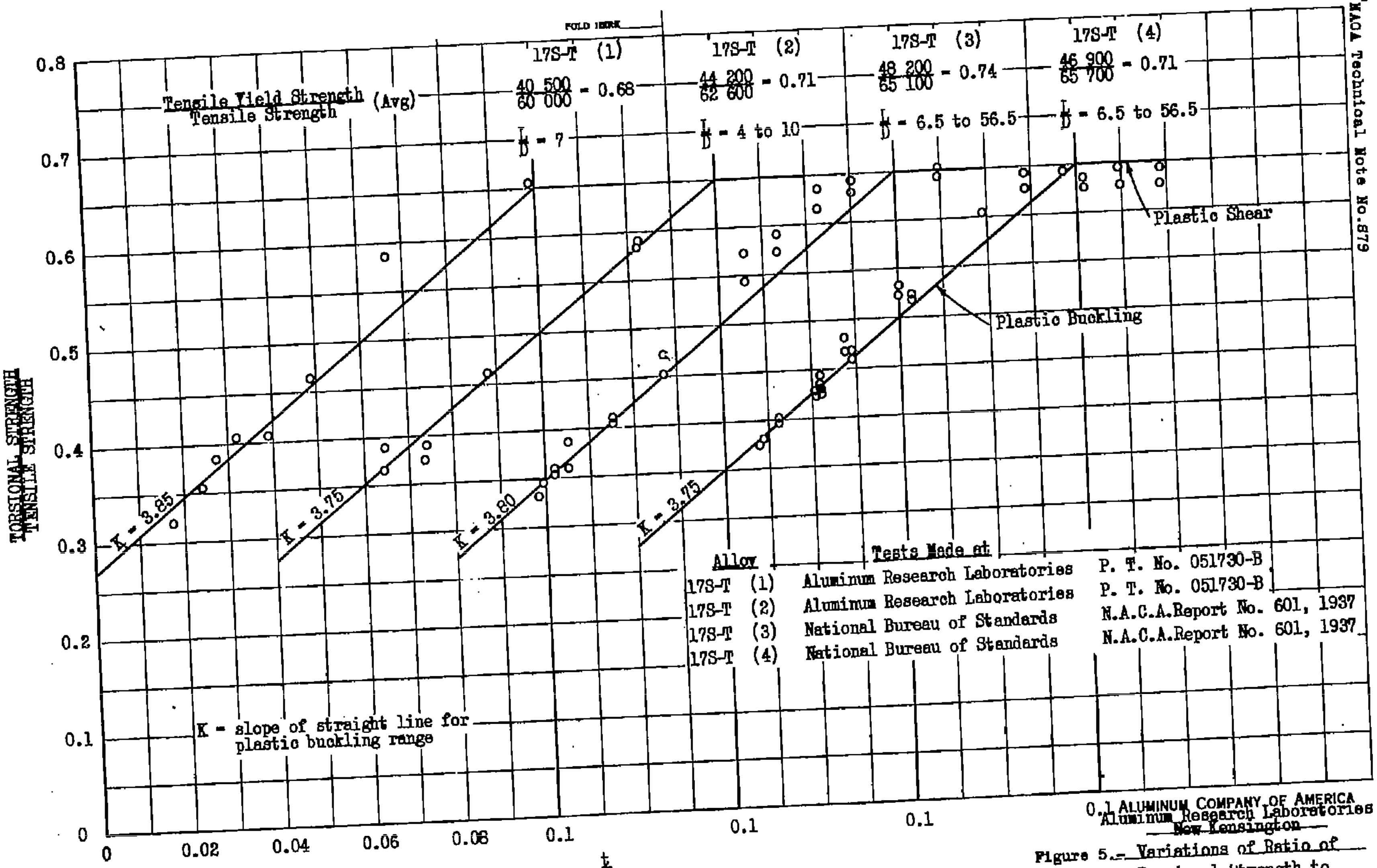

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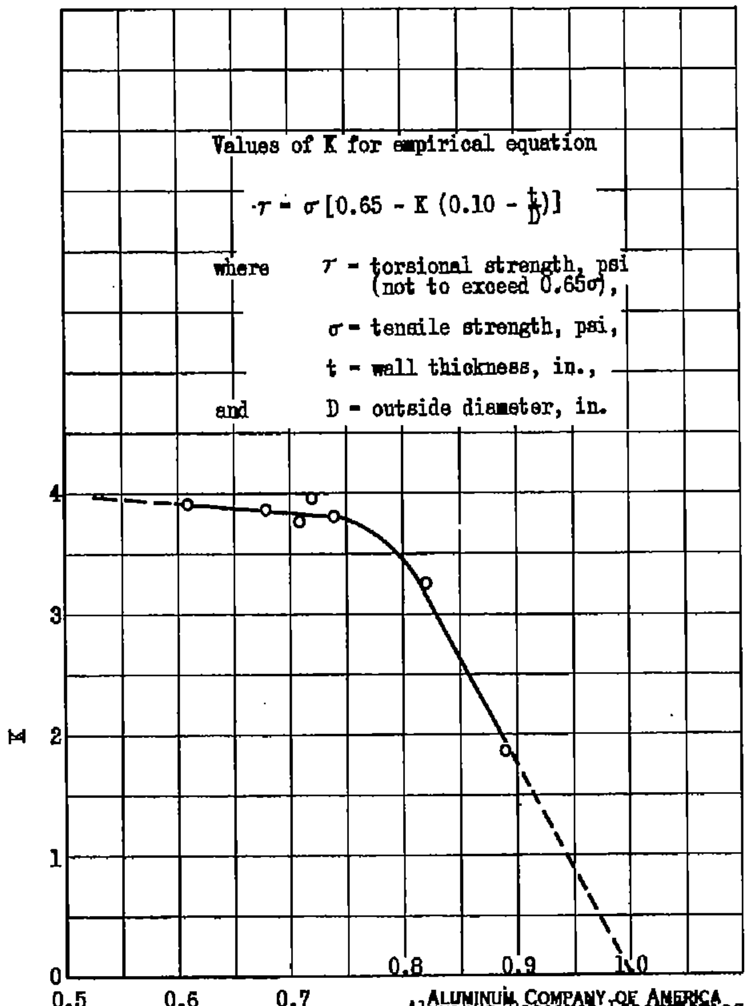
Figure 5.- Variations of Ratio of Torsional Strength to Tensile Strength with $\frac{t}{D}$

PHYSICAL TEST NO. As Noted

KIND OF METAL As Noted

PLOTTED ... DATE ... APPROVED ... DATE ...



TENSILE YIELD STRENGTH / TENSILE STRENGTH

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Fig.6 Variation of K with Ratio of Tensile Yield to Ultimate Strength

PHYSICAL TEST NO.

KIND OF METAL Aluminum Alloy

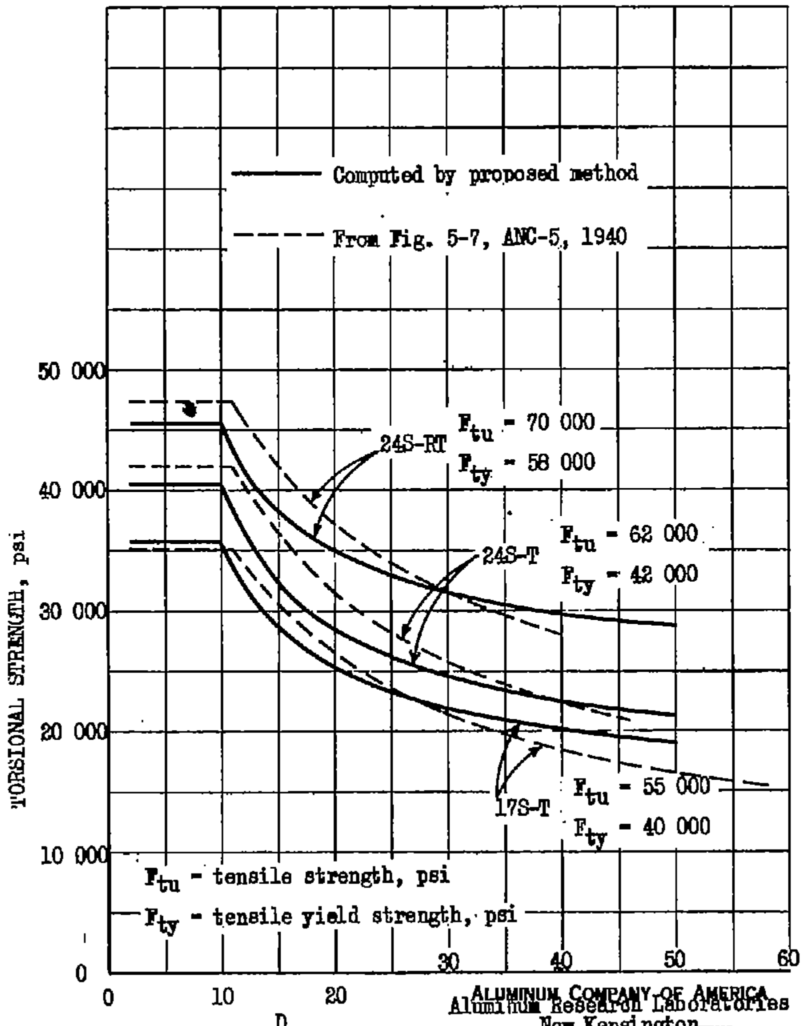

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Figure 9.- Variations of Torsional Strength with $\frac{D}{t}$ for Tubing Having Specified Minimum Properties

PHYSICAL TEST NO.

KIND OF METAL As Noted

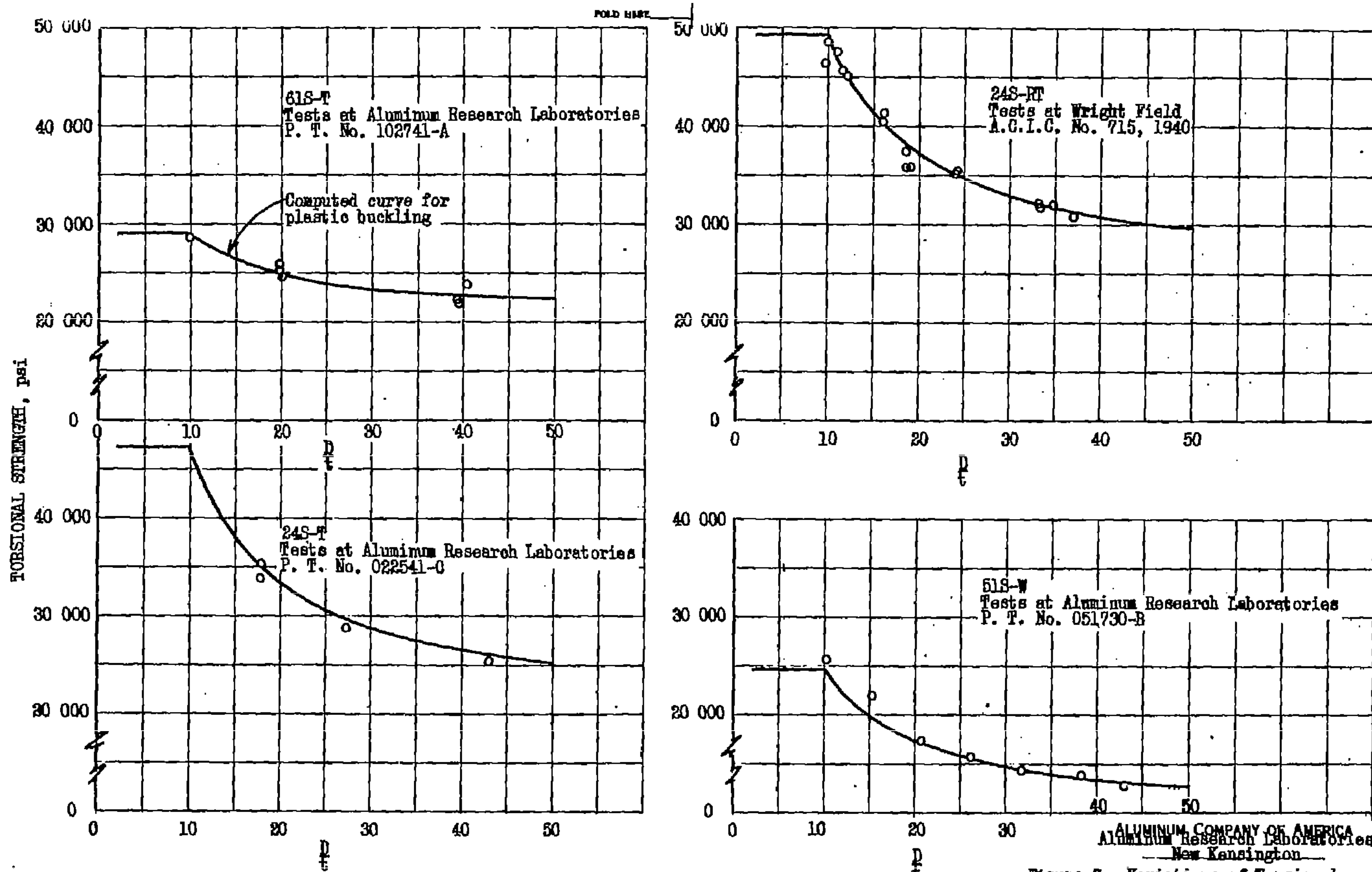

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Figure 7.- Variations of Torsional Strength with $\frac{D}{t}$

Physical Test No. As Noted
Kind of Metal As Noted

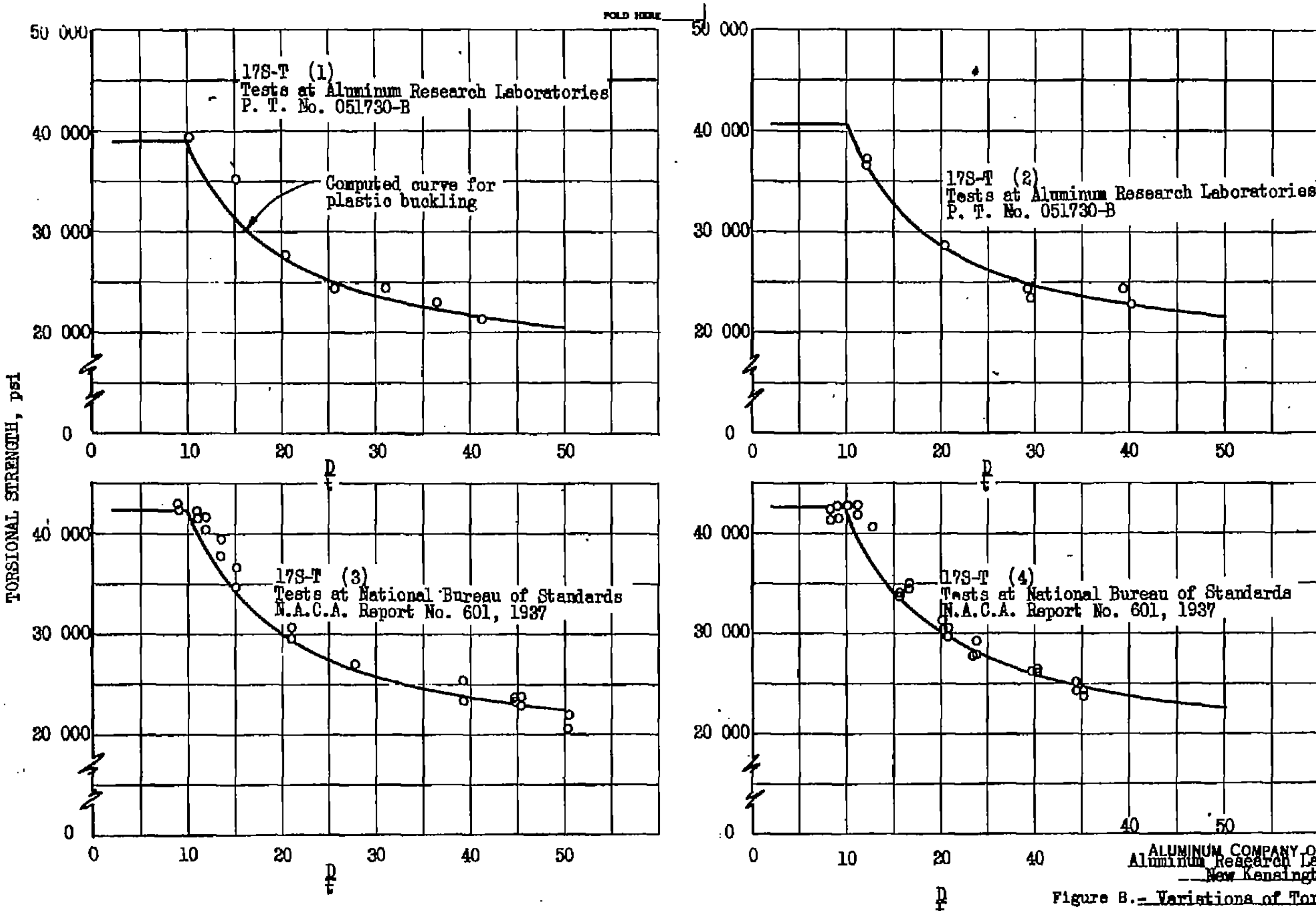

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Figure 8.- Variations of Torsional Strength with $\frac{D}{t}$

PHYSICAL TEST NO. As Noted

KIND OF METAL As Noted

Fig. 8

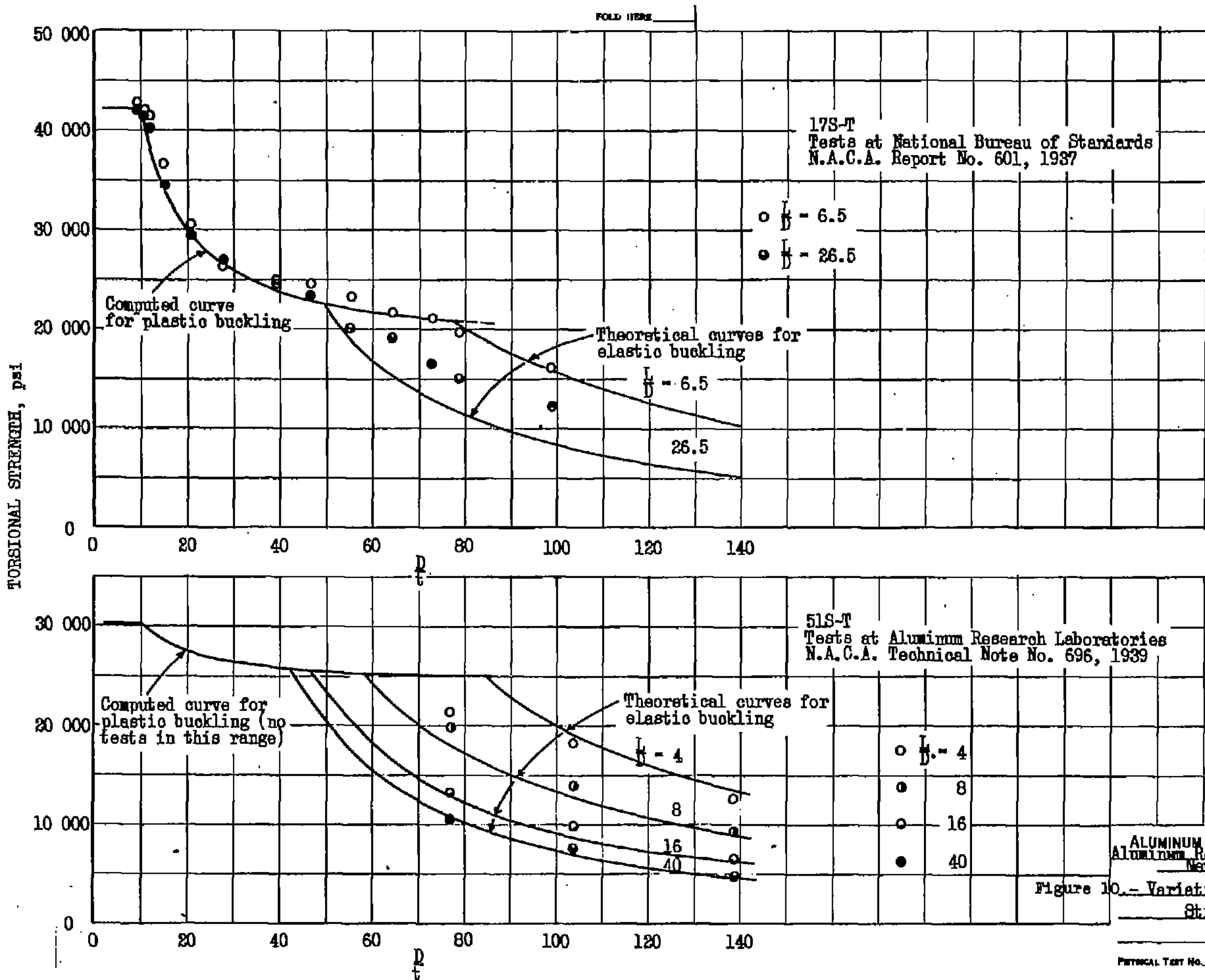

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Figure 10.- Variations of Torsional Strength with $\frac{D}{t}$

Physical Test No.

Kind of Metal As Noted

Plotted ___ Date ___ Approved ___ Date ___